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# CALCULATION OF THE EFFECTIVE PERMITTIVITY OF A TWO-PHASE STREAM

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A calculating equation is proposed for the effective permittivity of bubbly and gas–drop streams.

The methods of electrical conductivity and inductance [1], which are widespread in the diagnostics of two-phase flows, cannot be used to measure the volumetric content of the gaseous phase in a stream of dielectric liquid. In this connection one can use a capacitive method based on measuring the capacitance of a capacitor placed in the two-phase stream.

The dependence of the effective permittivity of a two-phase stream on the volumetric content of the disperse phase in it will be decisive for the use of this method. This dependence must also be at hand in many calculations of electron–ion technology and in problems of the electrohydrodynamics of two-phase flows [2, 3].

We note that methods are known in the literature [4-6] for calculating the coefficients of effective conductivity of heterogeneous (nonflowing) media. Unfortunately, they ignore the possibility of reorganization of the structure of a two-phase stream with an increase in the volumetric content of the disperse phase. For example, the change in the mode of flow of a bubbly stream has a crisis character, so that the coefficients of conductivity should undergo a discontinuity at some limiting attainable volumetric bubble content.

Let us consider a disperse stream of two dielectric media. Let the disperse phase be present in the form of equal-sized spherical drops or bubbles and be characterized by a permittivity $\varepsilon_2$, while the carrier (dispersion) phase is characterized by a permittivity $\varepsilon_1$. We assume that the fluctuations in volumetric content, number density, and sizes of the disperse particles caused by turbulent pulsations and processes of particle fragmentation and coalescence do not exceed their average values $\varphi$, N, and R by many times.

Let a small plane capacitor be placed in the two-phase stream so that the functions $\varphi$, N, and R can be taken as uniform in the space between its plates. At the same time, the volume of the capacitor is representative, i.e., $a\mathrm{bh} \gg R^3$, so that the nonuniform electrostatic field due to the disperse particles can be averaged.

Let the distance $a$ between the capacitor plates be a multiple of 2R. We divide up the region of the two-phase stream in it into layers of thickness 2R by equipotential planes. In the resulting system of $a/2R$ series-

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connected elementary capacitors some will contain disperse particles (conditionally "two-phase" capacitors) while others will not ("one-phase" capacitors). The number of elementary "two-phase" capacitors will be $a\sqrt[3]{N}$ while the number of "one-phase" elementary capacitors is $a/2R - a\sqrt[3]{N}$.

The capacitance of an elementary "one-phase" capacitor is

$$\Delta C_1 = \frac{\varepsilon_1}{4\pi}\frac{bh}{2R},$$

while the capacitance of the system of elementary "one-phase" capacitors is determined from the condition of their series connection,

$$C_1 = \frac{\varepsilon_1}{4\pi}\frac{bh}{a}\frac{1}{1-2R\sqrt[3]{N}}. \tag{1}$$

To calculate the capacitance of an elementary "two-phase" capacitors we assume that its capacitance is unchanged if the spherical disperse particles are replaced by cylindrical ones [7]. In this case the capacitance of an elementary "two-phase" capacitor can be found as the sum of the capacitances of two "one-phase" capacitors with permittivities $\varepsilon_1$ and $\varepsilon_2$:

$$\Delta C_{12} = \frac{hb}{4\pi}\frac{\varepsilon_1(1-\pi R^2\sqrt[3]{N^2})+\varepsilon_2\pi R^2\sqrt[3]{N^2}}{2R}.$$

The capacitance of the system of all elementary "two-phase" capacitors is determined from the condition of their series connection and will be

$$C_{12} = \frac{bh}{4\pi a}\frac{\varepsilon_1(1-\pi R^2\sqrt[3]{N^2})+\varepsilon_2\pi R^2\sqrt[3]{N^2}}{2R\sqrt[3]{N}}. \tag{2}$$

The capacitance of this capacitor is found using Eqs. (1) and (2) and is

$$C = \frac{bh}{4\pi a}\frac{\dfrac{\varepsilon_1}{1-2R\sqrt[3]{N}}\left(\dfrac{1}{2}\varepsilon_2\pi R\sqrt[3]{N}+\dfrac{1-\pi R^2\sqrt[3]{N^2}}{2R\sqrt[3]{N}}\right)}{\dfrac{\varepsilon_1}{1-2R\sqrt[3]{N}}+\dfrac{1}{2}\varepsilon_2\pi R\sqrt[3]{N}+\varepsilon_1\dfrac{1-\pi R^2\sqrt[3]{N^2}}{2R\sqrt[3]{N}}}. \tag{3}$$

On the other hand, the same capacitance can be calculated through the effective permittivity of the two-phase stream,

$$C = \frac{\varepsilon_{ef}}{4\pi}\frac{bh}{a}. \tag{4}$$

Comparing Eqs. (3) and (4) and allowing for the relation $\varphi = (4\pi/3)R^3N$, after transformations we find

$$\varepsilon^* = \frac{\varepsilon_{ef}}{\varepsilon_1} = \frac{1}{1+\dfrac{3}{2}\dfrac{(1-\varepsilon_2/\varepsilon_1)\varphi}{1-\sqrt[3]{\dfrac{9\pi}{16}}\left(1-\dfrac{\varepsilon_2}{\varepsilon_1}\right)\varphi^{2/3}}}. \tag{5}$$

If we make the limiting transition to a one-phase stream in (5) then we obtain $\varphi \to 0$ and $\varepsilon^* \to 1$. Moreover, it follows from Eq. (5) that for any $\varphi \in [0, 1]$ and two dielectrically indistinguishable liquids $\varepsilon_1 = \varepsilon_2$ $\varepsilon^* = 1$, which agrees with well-known physical concepts.

In bubbly and gas–drop streams $\varphi$ cannot exceed the value of $\pi/6$ determined by the condition of closest packing of spheres. For a gas–drop stream with $\varepsilon_2 > \varepsilon_1$ it follows formally from (5) that for $\varphi = \varphi_{cr}^{(2)} > \pi/6$, where $\varphi_{cr}^{(2)}$ satisfies the cubic equation

$$\varphi - \sqrt[3]{\frac{\pi}{6}}\varphi^{2/3} - \frac{2}{3}\frac{1}{\varepsilon_2/\varepsilon_1 - 1} = 0,$$

$\varepsilon^* \to \infty$, which doesn't make physical sense. And for a bubbly stream with $\varepsilon_1 > \varepsilon_2$ we can find from Eq. (5) that for values of

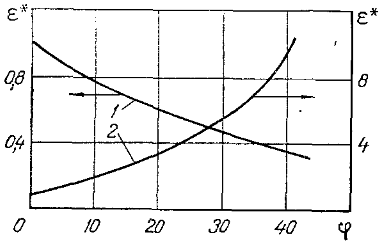


**Fig. 1.** Reduced effective permittivity of a stream as a function of the volumetric content $\varphi$(%) of: 1) bubbles of the gaseous phase; 2) aerosol of the liquid phase.

$$\varphi = \varphi_{cr}^{(1)} = \frac{4}{3\sqrt{\pi}} \sqrt{\frac{1}{(1-\varepsilon_2/\varepsilon_1)^3}} > \frac{\pi}{6},$$

$\varepsilon^* = 0$, which has no meaning.

One can show, for example, that the values $\varphi_{cr}^{(1)}$ determine the conditions under which the capacitance of all the "two-phase" capacitors is formally reduced to zero, in which case the surface gas content $\varphi_s$ of the bubbles in the layers becomes greater than unity, which violates the definition of $\varphi_s$,

$$\varphi_s = \frac{\pi R^2 bh \sqrt[3]{N^2}}{bh} = \frac{\varepsilon_1}{\varepsilon_1 - \varepsilon_2} > 1,$$

since $\varepsilon_1 > \varepsilon_2$.

Thus, for bubbly and aerosol streams Eq. (5) will be correct only for $\varphi \leq \pi/6$.

The function (5) can be used to find the volumetric content of the disperse phase from the measured capacitance of a capacitor in a two-phase stream. For this we note that $\varepsilon^* = C^* = C(\varphi)/C(0)$, where $C(\varphi)$ is the capacitance of the capacitor containing a two-phase dielectric while $C(0)$ is the capacitance of the same capacitor filled with a one-phase dielectric with a permittivity $\varepsilon_1$. Using this, we write (5) in the form of a cubic equation (substituting $\varphi = t^3$)

$$\varphi + a\varphi^{2/3} + c = 0,$$

the real root of which can be found using the Cardan solution,

$$\sqrt[3]{\varphi} = \sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{P}{3}\right)^3 + \left(\frac{q}{2}\right)^2}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\left(\frac{P}{2}\right)^3 + \left(\frac{q}{2}\right)^2}} - \frac{1}{3}a, \tag{6}$$

where

$$a = \frac{2}{3}\sqrt[3]{\frac{9\pi}{16}}\,\frac{1-C^*}{C^*}; \; p = -\frac{1}{3}a^2;$$

$$q = 2\left(\frac{a}{3}\right)^3 + c; \; c = -\frac{2}{3}\,\frac{1-C^*}{C^*}\,\frac{\varepsilon_1}{\varepsilon_1 - \varepsilon_2}.$$

It should be noted that the capacitance $C(\varphi)$ must be measured at several values of the electrostatic field strength so as not to cause its reaction to the gas content $\varphi$ in the capacitor. Another drawback of the capacitive method is its high sensitivity to the geometry of the disperse particles, so that Eq. (5) is valid only for monodisperse streams. A departure from spherical particle shape results in the need to use capacitive detectors of large size.

Graphs of the function $\varepsilon^* = \varepsilon^*(\varphi)$ calculated from Eq. (5) for bubbly and gas–drop streams of water–air and air–water systems are shown in Fig. 1. Evidently, the graphic method is more convenient to use than the cumbersome solution (6) to find $\varphi$.

We note that in bubbly streams $\varepsilon_1 \gg \varepsilon_2$, so that the function (5) can be written as

$$\varepsilon^* = \frac{\varepsilon_{ef}}{\varepsilon_1} = \left(1 + \frac{3}{2}\,\frac{\varphi}{1 - \sqrt[3]{\frac{9\pi}{16}}\varphi^{2/3}}\right)^{-1}, \tag{7}$$

while in gas–drop streams $\varepsilon_2 \gg \varepsilon_1$ and Eq. (5) takes the form

$$\varepsilon^* = \frac{\varepsilon_{ef}}{\varepsilon_1} = \left(1 - \frac{3}{2}\frac{\varphi\varepsilon_2/\varepsilon_1}{1+\sqrt[3]{\frac{9\pi}{16}\frac{\varepsilon_2}{\varepsilon_1}\varphi^{2/3}}}\right)^{-1}.$$

The variation of the effective permittivity of a two-phase stream containing electrically conducting disperse particles can also be of interest. This variation can be obtained from (5) by seeking the simple limit $\lim_{\varepsilon_2/\varepsilon_1 \to \infty} \varepsilon^*(\varphi, \varepsilon_2/\varepsilon_1)$.. As a result, we obtain

$$\varepsilon^* = \frac{\varepsilon_{ef}}{\varepsilon_1} = \frac{1}{1-\sqrt[3]{\frac{6}{\pi}\varphi}}.$$

From this it is seen that $\varepsilon^* \to \infty$ at $\varphi = \pi/6$. Physically this means that electrically conducting spheres in closest packing form conducting clusters [6] and the permittivity assumes an infinite value, i.e., the two-phase stream becomes electrically conducting.

The equations presented describe the dependence on $\varphi$ of not only the effective permittivity but also the effective electrical conductivity of two-phase streams, as well as the effective thermal conductivity in the corresponding two-phase (nonflowing) media. For example, Eq. (7) coincides with the dependence on $\varphi$ of the effective electrical conductivity of a bubbly stream containing electrically nonconducting bubbles, which was verified experimentally in [7].

## NOTATION

$\varepsilon_1$, $\varepsilon_2$, permittivities of the carrier and disperse streams; $\varphi$, volumetric content of disperse phase; N, number density of disperse particles; R, radius of a spherical particle; $a$, distance between capacitor plates; b and h, dimensions of capacitor plates; $\Delta C_1$, capacitance of an elementary "one-phase" capacitor; $C_1$, capacitance of the system of elementary "one-phase" capacitors; $\Delta C_{12}$, capacitance of an elementary "two-phase" capacitor; $C_{12}$, their total capacitance; C, capacitance of the representative capacitor; $\varepsilon_{ef}$, effective permittivity of the two-phase stream; $\varepsilon^*$, its reduced permittivity.

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